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R – 2322

Reg. No. : .....

Name : .....

Fourth Semester B.Sc. Degree Examination, July 2023

First Degree Programme under CBCSS

Mathematics

Core Course III

MM 1441 : ELEMENTARY NUMBER THEORY AND CALCULUS II

(2018 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **all** questions :

1. Verify whether $100 \equiv -1 \pmod 3$.
2. Find the least residue of 10009 modulo 11.
3. Find the least positive integer x such that $x \equiv 0 \pmod{20}$.
4. Show that a palindrome with an even number of digits is divisible by 11.
5. State the Chinese remainder theorem.
6. Check whether $\int_0^1 xy^2 dx = \int_0^1 xy^2 dy$.
7. Define an inverse-square field.
8. State Green's theorem.

P.T.O.

9. Estimate $\int_0^2 \int_1^3 \int_1^2 xy^2 \, zdzdydx$.

10. Determine whether the vector field $F(x, y) = (y + x)i + (y - x)j$ is conservative on some open set.

(10 x 1 = 10 Marks)

PART – B

Answer **any eight** questions.

11. Show that $a \equiv b \pmod{m}$ if and only if a and b leave the same remainder when divided by m.

12. Find the remainder when 3^{247} is divided by 17.

13. Solve the linear congruence $12x \equiv 6 \pmod{7}$.

14. Using the Pollard $p - 1$ method, find a nontrivial factor of 2813.

15. Find the positive integers n for which $\sum_{k=1}^{n} k!$ is a square.

16. Evaluate : $\int_0^1 \int_{-2}^2 (x + y) \, dydx$.

17. Compute $\iint_R \sin\theta \, dA$, where R is the region in the first quadrant that is outside the circle $r = 2$ and inside the cardioid $r = 2(1 + \cos\theta)$.

18. Find the divergence and curl of $F(x, y, z) = x^2yi + 2y^3zj + 3zk$.

19. Estimate $\int_C (xy + z^3) \, ds$ from $(1, 0, 0)$ to $(-1, 0, \pi)$ along the helix C given by the parametric equations $x = \cos t$, $y = \sin t$, $z = t$ where $0 \leq t \leq \pi$.

20. Find the work performed by the force field $F(x, y, z) = x^2 i + 4xy^3 j + y^2 xk$, on a particle that traverses the rectangle C in the plane $z = y$.

21. Find the outward flux of the vector field $F(x, y, z) = zk$ across the sphere $x^2 + y^2 + z^2 = 1$.

22. Find the Jacobian of the transformation $u = x + y$ and $v = x - y$.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions.

23. Show that the linear congruence $ax \equiv b \pmod{m}$ is solvable if and only if $d \mid b$, where $d = (a, m)$. If $d \mid b$, then prove that it has d incongruent solutions.

24. Find the last digit in the decimal value of $2017^{2018^{2019}}$.

25. Solve the linear system $x = 1 \pmod 3$, $x \equiv 2 \pmod 4$, and $x \equiv 3 \pmod 5$.

26. State and prove the Wilson's theorem.

27. Evaluate $\iint_R xydA$, over the region R enclosed between $y = \frac{x}{2}$, $y = \sqrt{x}$, $x = 2$ and $x = 4$.

28. Compute $\int_C x^2 ydx + xdy$ where C is the closed triangular path OABO having vertices $O(0, 0)$, $A(1, 0)$ and $B(1, 2)$.

29. Show that $F(x, y) = 2xy^3 i + (1 + 3x^2 y^2) j$ is a conservative vector field on the entire xy-plane. Also find ϕ.

30. Find the work done by the force field $F(x, y) = (e^x - y^3) i + (\cos y + x^3) j$ on a particle that travels once around the unit circle $x^2 + y^2 = 1$, in the counter clockwise direction.

31. Using Stoke's theorem prove that $\int_C yzdx + zxdy + xydz = 0$ where C is the curve $x^2 + y^2 = 1$, $z = y^2$.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions.

32. (a) Let p be a prime and a any integer such that p does not divide a. Then show that the least residues of the integers a, 2a, 3a,.., $(p-1)$ a modulo p are a permutation of the integers $1, 2, 3, \ldots, (p-1)$.

(b) State and prove the Fermat's little theorem and hence estimate the remainder when 24^{1947} is divided by 17.

33. (a) Calculate $\iint_{\sigma} xz\,dS$ where σ is the part of the plane $x + y + z = 1$ that lies in the first octant.

(b) Suppose that a curved lamina σ with constant density $\delta(x, y, z) = \delta_0$ is the portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 1$. Find the mass of the lamina.

34. (a) Evaluate $\iint_R \frac{x-y}{x+y}\,dA$ where R is the region enclosed by $x - y = 0$, $x - y = 1$, $x + y = 1$ and $x + y = 3$.

(b) Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.

35. Verify Gauss divergence theorem over the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$ for the vector field

$$F(x, y, z) = (x^2 - yz)\,i + (y^2 - zx)\,j + (z^2 - xy)\,k$$

(2 × 15 = 30 Marks)